

Logic and Argumentation

Douglas E. HILL
Golden West College (United States of America)

Using Truth Tables to Teach Enthymemes

Abstract: A method is presented to teach enthymemes using truth tables. Students are given an argument with a missing step, some choices for the missing step, and are to put these into a truth table. They are then to use the criteria of consistency, validity, and charity to choose a step to complete the argument. This technique is intended to teach introductory logic students about enthymemes, and to give them more practice with truth tables and symbolization. This method is limited to sentence logic arguments, but can also evaluate enthymemes missing both a premise and conclusion.

Keywords: enthymeme, truth table, multiple choice, assumption, teaching logic, consistency, validity, charity

1. Introduction

Most arguments outside of logic class are enthymemes: arguments with an unstated premise or conclusion (or both). Yet enthymemes typically get short-shrift in logic classes and textbooks: they are taught with categorical logic if they are taught at all. But statement logic can also be used to teach introductory logic students about enthymemes (Pole 1980, 325). This connects logic class with reasoning outside of logic class and teaches about the roles assumptions play in common reasoning.

An arguer might not state a premise or a conclusion to save time: the step¹ seems so obvious, stating it would seem a waste of time. Or she might leave out a premise that the hearer might not know to flatter him that he must already know this. Or she might leave out a questionable premise hoping that the hearer will either assume that the premise must be obvious or ignore it altogether. Often this is unintentional, as the arguer

¹ This paper uses the term “step” throughout to refer to either a premise or a conclusion.

might not realize that someone else might question a premise that she finds so obvious. So finding an unstated premise (an assumption) can help one figure out what is wrong with a faulty argument.

Fortunately, finding missing steps is usually not too difficult. One is often able to leave steps out of an argument, and still be understood, because those steps are simple. Very often the missing steps are atomic statements or very simple compound statements. The key here is that enthymemes lend themselves well to multiple choice problems. Given an argument with an unstated step, my students often have a hard time articulating what the missing step must be. But given a few candidate steps, they can use this technique to determine which of the given choices (if any) could be the unstated step.

2. Truth Table Method: Three Requirements

To use this method, students need to be familiar with using truth tables *both* to evaluate arguments for validity *and* to evaluate sets of statements for consistency. To find a missing step (premise or conclusion), the students are to put the candidate steps into the truth table along with the stated steps, and pick the step that satisfies three requirements: *consistency*, *validity*, and *charity*.²

1) *Consistency*: The added step is consistent with the stated steps. That is, there is a truth value assignment for which all stated and added steps are true.

2) *Validity*: When the step is added, the argument is valid. That is, there is no (longer) a row in the truth table in which all the premises are true but the conclusion is false.

² Dale Jacquette (1996, 2) suggests four requirements for the principle of charity: adding minimal steps to yield a “valid, sound, nonredundant, [and] noncircular inference.” *Validity* is already one of my three requirements. Part of soundness is included in my requirement of *consistency*, since an argument with inconsistent premises would be valid but unsound. However, requiring the found step to always be true is too strong a requirement: we want to be able to find a false assumption. Under my method, *charity* is a separate requirement, one that asks students to use their judgment beyond what a truth table can tell them. “*Noncircular*” would be a good fourth requirement: that the truth table column for a found premise be different than the given conclusion (and vice-versa). But in the interest of simplicity, I discuss the validity of begging the question elsewhere in my course and avoid it as a choice when writing problems. I also avoid redundancy in the choices I offer. Such choices would be uncharitable, so “nonredundant, and noncircular” remain within the principle of charity for my method.

3) *Charity*: The added step (and resulting argument) can be charitably attributed to the arguer.

The first requirement prevents adding a premise (such as the negation of an earlier premise) that would create an inconsistent set of premises. Such an argument would be valid but unsound, so such a step should not be intended in an argument meant to be persuasive. In fulfilling the second requirement, the found step makes a previously invalid argument valid. If the argument were already valid, there would be no invalidating truth-value assignment to begin with.³

Evaluating the first two requirements can be done with truth tables alone; the third (charity) requires some outside knowledge. We might like to make the third condition stronger, requiring that an added premise be known to be true by both arguer and hearer. However, the added premise is often not known to be true by both, and may not be true at all. And finding an untrue or unshared premise is one of the most useful results of analyzing enthymemes. But we often can use our outside knowledge and our understanding of the argument to come up with some candidate premises or conclusions, and then use the truth table method to confirm or eliminate these possible steps.

2.1. Assumed Premises

For example, consider the following argument.⁴

1. If gasoline is expensive, we should carpool. Thus, we should carpool. (G: Gasoline is expensive. C: We should carpool.)

Using the given symbolization, this argument can be written as:

2. $G \rightarrow C \therefore C$.

The symbolized argument is so obviously flawed that it would be uncharitable to assume that this is the argument that the author intended. More likely, she assumed a premise. Often the hidden step will be obvious. However, it is often not so obvious to students, so I will offer them three possible steps to consider, in this case: $\sim G$, $G \bullet \sim C$, and G . Then they should put each candidate premise in a truth table between the given premises and the conclusion, as follows:

³ Some of the homework problems that I give my students are already valid arguments as stated, so that students are supposed to choose no “additional step is necessary” instead of one of the offered steps.

⁴ I follow the convention (Layman 2005, ch. 6) of providing the symbolization after the statements. Students often have a hard time coming up with their own symbolizations, and providing this makes the problems uniform and thus easier to grade.

G	C	G	\rightarrow	C	\sim	G	,	G	\bullet	\sim	C	,	G	\therefore	C
T	T		T		F				F				T		T
T	F		F		F				T				T		F
F	T		T		T				F				F		T
F	F		T		T				F				F		F

We can now evaluate each candidate premise using the three requirements above. The first candidate premise, $\sim G$, fails the validity requirement while the second, $G \bullet \sim C$, fails the consistency requirement. The third candidate, G , puts a T in line 1 (making it consistent) and an F in line 4 (making the argument valid). If students can see which statement was most likely to be the missing premise, I encourage them to start with that statement. If they turn out to be correct, they may then skip evaluating the other candidate statements.

The last step requires that we put the assumption, G , into English form, and then evaluate the found step and the resulting argument. (This also gives students practice in going from symbols back into English.) Here, the missing step is “Gasoline is expensive”. The arguer appears to believe that this is an assumption she can share with her reader. (Even if one disagrees, this is a reasonable assumption to attribute to the arguer.)

In writing a well-posed problem, the stated premises need to be consistent with the conclusion, but should not (normally) validly imply it. That is, the truth table should yield at least two lines where every stated premise is true: at least one line where the conclusion is true (and thus consistent with the stated premises), and at least one line where the conclusion is false (making the argument, as stated, invalid). For the students to see how enthymemes actually work, this choice should be a natural assumption, which is usually the simplest one that fulfills the three requirements.

2.2. Implied Conclusions

Students can use the same three requirements to find a missing conclusion. But when finding a conclusion that follows from the premises (requirement 2), the conclusion will normally also be consistent with the premises (requirement 1).⁵ This makes missing conclusion problems a bit easier. For example, consider the following enthymeme:

⁵ If the premises are inconsistent to begin with, then the first condition cannot be satisfied. Such an argument cannot be salvaged by finding a missing step.

3. You'll like green tea ice cream only if you like green tea. But you don't like green tea! (I: You will like green tea ice cream. G: You like green tea.)

The arguer seems to have a conclusion she intends to communicate, yet she does not state it. Putting her premises in symbolic form (and warning students about "only if") yields:

4. $I \rightarrow G, \sim G$

Students reading this enthymeme can discern the intended point. But let us offer them the following choices: I, $\sim I$, and $I \bullet G$. We can then use a truth table to find or confirm the implied conclusion. We will put the candidate conclusions to the right of the premises in the truth table (as follows), and then see which one follows validly from the premises.

I	G	$I \rightarrow G$	$\sim G$	$\therefore I$	$\sim I$	$I \bullet G$
T	T	T	F	T	F	T
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	F	T	F

The only candidate conclusion that follows validly from the premises is $\sim I$ (as many students anticipate). I then ask my students to write the conclusion in unambiguous, natural English (which they find surprisingly difficult). They often closely follow the structure of the compound statement and write it as "It is not the case that you will like green tea ice cream." But I encourage them to write it in more natural English as "You won't like green tea ice cream." By not stating this conclusion, the arguer forced the hearer to take a simple logical step, *modus tollens*, and draw the conclusion in his own mind. Had she simply stated the conclusion for him, he might have ignored the structure of her argument. Had she done so, he would be less likely to be convinced.

As with missing premise problems, there should be at most one correct choice,⁶ and it should appear to be the natural conclusion of the argument. So, in the previous problem, $\sim I$ should be the correct choice, and neither $\sim I \bullet \sim G$ nor $\sim G \rightarrow \sim I$ should be offered (even though they also follow validly from these premises).

⁶ One can occasionally assign a problem in which none of the choices follow validly from the premises, so that the correct answer is "none of the above."

2.3. Symbol-Only Enthymeme Problems?

We often give our students arguments in symbol-only form to evaluate for validity. This lets them practice their truth-table skills without having to worry about the pitfalls of accurately symbolizing English arguments, which many students find particularly difficult. So too we can give them invalid symbolic arguments and ask them to find a premise that, if added, would make the argument valid. And we can ask them to find which symbolic statement would follow validly from a list of others. We can also assign such problems to teach or reinforce properties of the truth-functional connectives.

Missing premise problems would be done the same way as the problem above, except that only the consistency and validity requirements would need to be applied. (There is no need or ability to evaluate the arguer charitably.) For example, consider the argument $J \therefore \sim(J \rightarrow K)$, with possible choices: $\sim(K \rightarrow J)$, K , or $\sim K$ for the missing premise. Putting this problem in a truth table, using the method above, yields:

J	K	J	$\sim(K \rightarrow J)$	K	$\sim K$	$\therefore \sim(J \rightarrow K)$
T	T	T	F	T	F	F
T	F	T	F	F	T	T
F	T	F	T	T	F	F
F	F	F	F	F	T	F

The first candidate premise fails the consistency requirement; the second fails validity. The third candidate premise, $\sim K$, is both consistent with the stated steps, results in a valid argument when added, and is thus the correct answer.

However, finding a hidden premise depends in part upon the context of the argument, so it is not clear that symbol-only arguments can be called enthymemes. We should teach our students the logical principle that one always *can* make an argument valid by adding a premise, along with the caveat that one often *should not* do this. Some arguments are simply invalid and should not be saved by finding a missing premise.⁷

This is less of a concern when trying to find an unstated conclusion. Even without the context of an English argument, some statements will follow validly from any consistent set of statements, and some will not. Such problems may not be enthymemes, but there should be no confusion

⁷ I would like to thank Heather Battaly for raising these concerns.

in asking our students to find which statement follows from another set of statements. For example, students given premise L (a single atomic statement) with candidate conclusions: $L \rightarrow M$, $M \rightarrow L$, and $L \leftrightarrow M$ should be able to create the following truth table:

L	M	L	$\therefore L \rightarrow M$	$, M \rightarrow L$	$, L \leftrightarrow M$
T	T	T	T	T	T
T	F	T	F	T	F
F	T	F	T	F	F
F	F	F	T	T	T

Only the second candidate conclusion, $M \rightarrow L$, follows validly from the premise. This problem illustrates that any material conditional with a true consequent must be true.

3. Finding Both a Hidden Premise and a Hidden Conclusion

These methods can also evaluate enthymemes missing more than one step. The trick is either to figure out the missing assumption, and then treat it as a missing conclusion problem, or to figure out the point of the argument (the conclusion), and then treat it as a missing assumption problem.

In a famous example (Copi and Cohen 2009, 19; Socher 2007, 112-113), Abraham Lincoln (1864, 1) wrote:

5. “If slavery is not wrong, nothing is wrong.” (S: Slavery is wrong. N: No act is morally wrong.)

Lincoln’s point went beyond the mere conditional ($\sim S \rightarrow N$): he was asking his reader to reason from both his statement and common knowledge, so his sentence forms an argument with an unstated premise and an unstated conclusion.

Suppose the students are given choices S, N, $\sim S$, and $\sim N$ for the unstated premise and conclusion. Students may be able to see that N is neither true nor something that could be attributed to Lincoln. But this is a hint that $\sim N$ would be a safe assumption. The conclusion S is consistent and follows validly, and this argument can charitably be attributed to Lincoln.

Students need to be reminded of the charity requirement, as some will choose $\sim S$ for the assumed premise, and N for the conclusion. The resulting argument meets the requirements of consistency and validity, but it would be uncharitable to attribute such an argument to Lincoln. Many enthymemes have the structure of modus ponens, but not this one.

Students can often avoid this problem by starting with the conclusion. Students readily intuit Lincoln's point to be that slavery is wrong. Making S the conclusion, $\sim N$ readily follows as the assumed premise.

4. Conclusion

This method for finding missing steps provides a quick and simple way to use multiple choice to teach about enthymemes in an introductory logic or critical thinking class. Truth tables are limited to deductive arguments, are too cumbersome to handle complicated arguments, but provide a useful tool to teach about validity. The truth table method above shares these properties. Despite these limitations, truth tables are useful pedagogical tools that can be expanded to teach about enthymemes.

Enthymemes need not be limited to categorical syllogisms in an introductory logic class; we can also use truth tables to teach about enthymemes. We can give the students some enthymemes and then give them some choices for the missing steps. For sentence logic arguments, they can add candidate steps to the truth table, and then figure which missing step is consistent with the stated steps and makes the argument valid. For enthymemes missing both a premise and a conclusion, students should either figure out the assumption (a premise that can be charitably attributed to the arguer) or the point (the unstated conclusion), and then proceed as with a missing conclusion or a missing premise problem, respectively. These methods let the students apply the principle of charity: they need to confirm that the added steps and resulting arguments can charitably be attributed to the arguer.

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